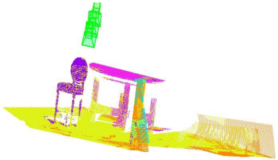
Point Clouds: Sensors and Algorithms

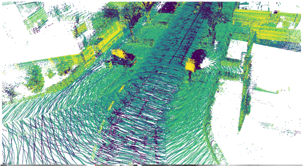
Steven A. Parkison

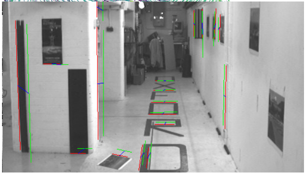
November 14, 2022

Overview

1. Sensors
2. Point Cloud Algorithms
3. Current Research







Sensors

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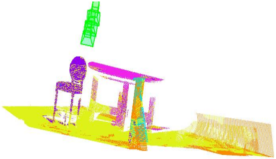
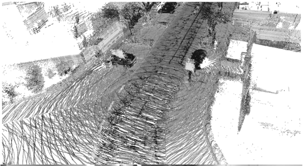
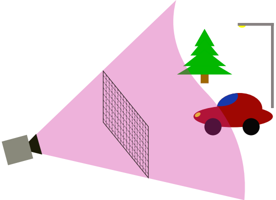
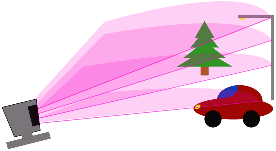
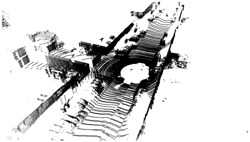




3D Sensor Registration







Rotating Scanning LIDAR

source: Wikimedia

Rotating Scanning LIDAR

Surround Sensors[6]
(mid to long range)

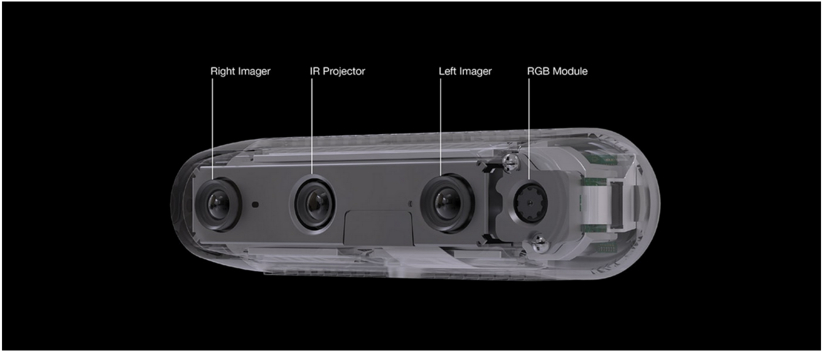
Sensor	HDL-64E	HDL-32	Puck	Puck LITE	Puck Hi-Res	Puck 32MR	Ultra Puck	Alpha Puck
								
Range	Up to 120m	Up to 100m	100m	100m	100m	120m	200m	Up to 300m[5]
Range Accuracy	Up to ±2 cm (Typical)[4]	Up to ±2 cm (Typical)[1]	Up to ±3 cm (Typical)[1]	Up to ±3 cm (Typical)[1]	Up to ±3 cm (Typical)[1]	Up to ±3 cm (Typical)[1]	Up to ±3 cm (Typical)[1]	Up to ±3 cm (Typical)[1]
# of Lines	64	32	16	16	16	32	32	128
Horizontal FoV	360°	360°	360°	360°	360°	360°	360°	360°
Vertical FoV	26.9°	41.33°	30°	30°	20°	40°	40°	40°
Horizontal Resolution	0.08° – 0.35°	0.1° – 0.4°	0.1° – 0.4°	0.1° – 0.4°	0.1° – 0.4°	0.1° – 0.4°	0.1° – 0.4°	0.1° – 0.4°
Vertical Resolution	0.4°	1.33°	2.0°	2.0°	1.33°	0.33° (min)	0.33° (min)	0.11° (min)
Points Per Second (Single Return Mode)	~ 1,300,000	~ 695,000	~ 300,000	~ 300,000	~ 300,000	~ 600,000	~ 600,000	~ 2,400,000
Points Per Second (Dual Return mode)	~ 2,200,000[5]	~ 1,390,000	~ 600,000	~ 600,000	~ 600,000	~ 1,200,000	~ 1,200,000	~ 4,800,000
Refresh Rate	5-20 Hz	5-20 Hz	5-20 Hz	5-20 Hz	5-20 Hz	5-20 Hz	5-20 Hz	5-20 Hz
Operating Voltage	12V - 32V	9 V – 18 V	9 V – 18 V	9 V – 18 V	9 V – 18 V	10.5 V – 18 V	10.5 V – 18 V	9 V – 28 V
Power Consumption	60 W (Typical)[2]	12 W (Typical)[2]	8 W (Typical)[2]	8 W (Typical)[2]	8 W (Typical)[2]	10 W (Typical)[2]	10 W (Typical)[2]	< 30 W (Typical)
Weight (without cabling)	~ 28 lbs. (12.7 Kg)	~1.0 kg	~830 g	~590 g	~830 g	~925 g	~925 g	~3.5 kg
Operating Temp	-10°C to +60°C[3]	-10°C to +60°C[3]	-10°C to +60°C[3]	-10°C to +60°C[3]	-10°C to +60°C[3]	-20°C to +60°C[3]	-20°C to +60°C[3]	-20°C to +60°C[3]
Storage Temp	-40°C to +85°C	-40°C to +105°C	-40°C to +105°C	-40°C to +105°C	-40°C to +105°C	-40°C to +85°C	-40°C to +85°C	-40°C to +85°C
Output	UDP packets over Ethernet	UDP packets over Ethernet	UDP packets over Ethernet	UDP packets over Ethernet	UDP packets over Ethernet	UDP packets over Ethernet	UDP packets over Ethernet	UDP packets over Ethernet
Ethernet Connection	100 Mbps	100 Mbps	100 Mbps	100 Mbps	100 Mbps	100 Mbps	100 Mbps	1000 Mbps
GPS Timesync	$GPRMC	$GPRMC + $GPGGA	$GPRMC + $GPGGA	$GPRMC + $GPGGA	$GPRMC + $GPGGA	$GPRMC + $GPGGA	$GPRMC + $GPGGA	$GPRMC + $GPGGA
Laser	903nm Class 1 eye safe	903nm Class 1 eye safe	903nm Class 1 eye safe	903nm Class 1 eye safe	903nm Class 1 eye safe	903nm Class 1 eye safe	903nm Class 1 eye safe	903nm Class 1 eye safe
Water Resistance	IP67	IP67	IP67	IP67	IP67	IP67	IP67	IP67

source: Velodyne

RGBD Sensors

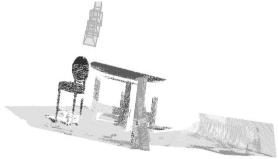
- *Stereo*
  Pros: Works indoors or outdoors. Just two cheap image sensors.
  Cons: Needs lighting. Computationally expensive to do stereo matching if its not done onboard.
- *Structured Light*
  Pros: Can work without lighting.
  Cons: Can be washed out by sunlight.
- *Flash LIDAR*
  Pros: Works indoors or outdoors, and in any lighting.
  Cons: The most expensive and power intensive option.

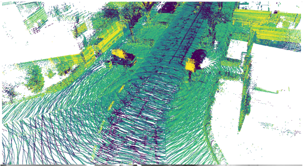
RGBD Sensors



source: Intel

Point Cloud Algorithms





Sensor Registration

Estimating the transformation of data from a sensor to:

prior data from the same sensor, producing an **egomotion** estimate;

data from another sensor, or the extrinsic **calibration** between sensors.

The Point Cloud Registration Problem

$$\mathcal{X} = \{\mathbf{x}_i\}, \mathbf{x}_i \in \mathbb{R}^3$$

Target Point Cloud The point cloud $\mathcal{X}_t$ we are trying to align to.

Source Point Cloud The point cloud $\mathcal{X}_s$ that we transform by $\mathbf{T} \in \mathrm{SE}(3)$ to align to the target point cloud.

$$\mathbf{T}\{\mathbf{x}\} \triangleq \mathbf{R}\mathbf{x} + \mathbf{p}$$
$$\mathbf{R} \in \mathrm{SO}(3)$$
$$\mathbf{p} \in \mathbb{R}^3$$

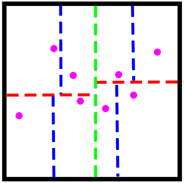
Iterative Closest Point (ICP) Algorithm, Besl and McKay

Step 1 Determine associations, $\mathcal{I}$, between target and source by finding the nearest neighbor.
Step 2 Given those associations, minimize the residuals, $\mathcal{R}$, between points.

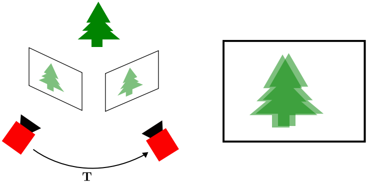
Step 1 Determine associations $\mathcal{I}$

$$i_k = \arg\min_k ||\mathbf{x}_k^t - \mathbf{T}\{\mathbf{x}_i^s\}||$$

$$\mathcal{I} = \{i_k\}$$



KD tree



Projective association

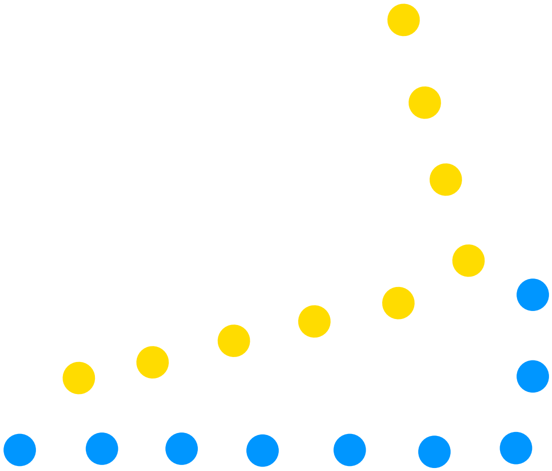
KD Tree Notebook

Step 2 Minimize residuals $\mathcal{R}$

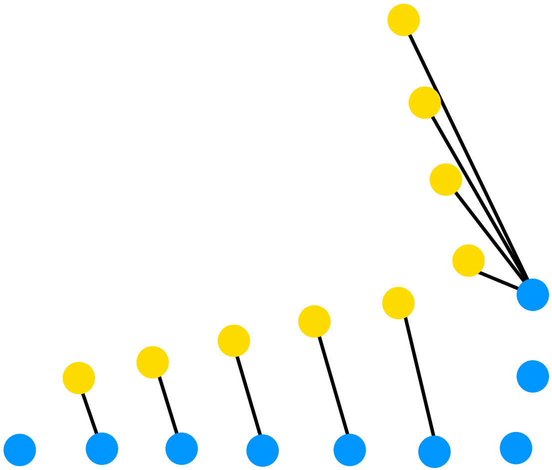
$$\begin{aligned} \mathbf{r}_k = \mathbf{x}_k^t - \mathbf{T}\{\mathbf{x}_k^s\} \\ \mathcal{R} = \{\mathbf{r}_k\} \end{aligned}$$

$$\underset{\mathbf{T}\in\mathrm{SE}(3)}{\arg\min} \sum_k ||\mathbf{r}_k||$$

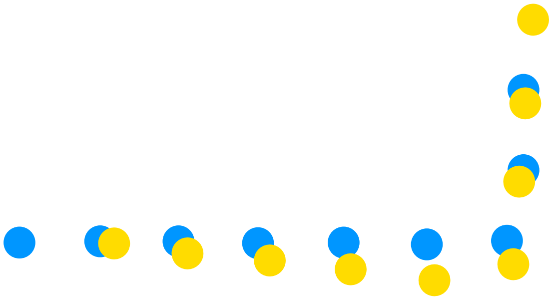
Iterative Closest Point (ICP)



Iterative Closest Point (ICP)



Iterative Closest Point (ICP)



ICP Notebook

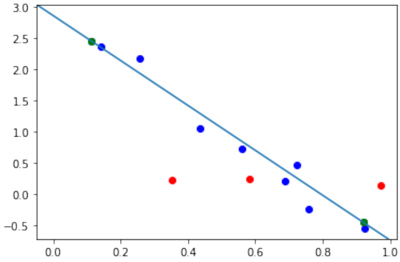
Random Sample Consensus

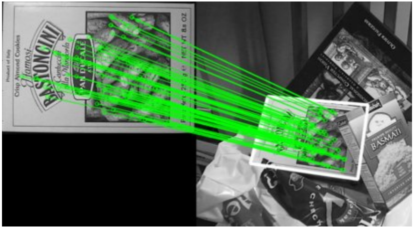
Problem: Fitting a model to a set of noisy data.
Solution: Iteratively follow a 3 step algorithm until a model is found.

Step 1 Randomly sample the minimum set of data, N, to fit model.
Step 2 Fit a model using the sampled data.
Step 3 Check remaining data to see inlier threshold (the termination criterium) is reached.

Random Sample Consensus

Good for fitting models of geometric primitives such as lines and planes. Or estimating transformations matrices in computer vision (such as homography or a stereo fundamental matrix)





Random Sample Consensus

RANSAC Notebook

Current Research

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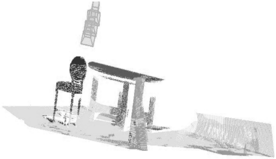
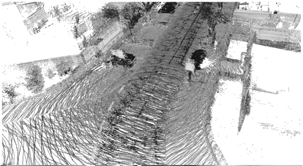
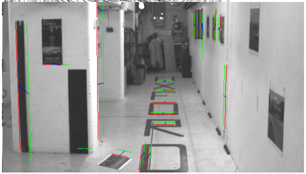


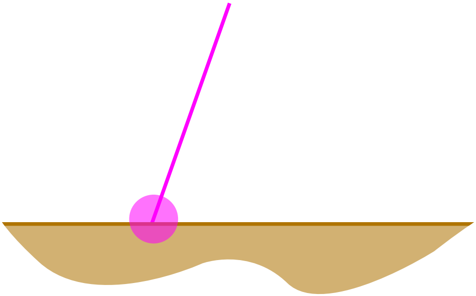




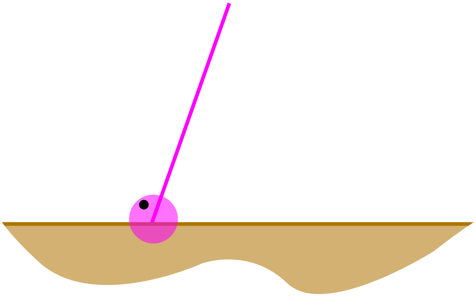
But do people still use ICP?

Generalized ICP, Segal et al.

Generalized ICP, Segal et al.

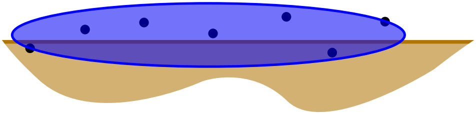


Generalized ICP, Segal et al.



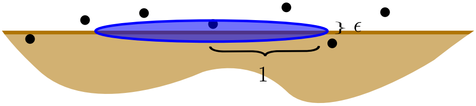
Generalized ICP, Segal et al.

$$\Sigma_{\mathbf{x}} = \sum_i^N \frac{(\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^\top}{N-1}$$

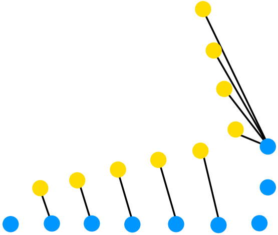


Generalized ICP, Segal et al.

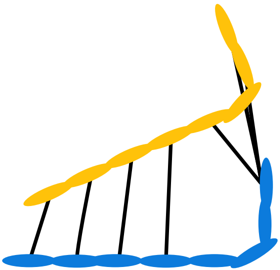
$$\Sigma_{\mathbf{x}} = Q\Lambda Q^\top$$
$$\lambda_1 = 1$$
$$\lambda_2 = \epsilon$$



Generalized ICP



Generalized ICP



Generalized ICP

$$-\log(p(\mathbf{r}_k|\mathbf{x}_k^t, \mathbf{x}_k^s, i_k; \mathbf{T})) = ||\mathbf{x}_k^t - \mathbf{T} \cdot \mathbf{x}_k^s||_{\Sigma_k^t + \mathbf{R}\Sigma_k^s\mathbf{R}^\top} + \text{const.}$$

Generalized ICP

$$-\log(p(\mathbf{r}_k|\mathbf{x}_k^t, \mathbf{x}_k^s, i_k; \mathbf{T})) = ||\mathbf{x}_k^t - \mathbf{T} \cdot \mathbf{x}_k^s||_{\Sigma_k^t + \mathbf{R}\Sigma_k^s\mathbf{R}^\top} + \text{const.}$$

$$\begin{aligned} f_{\texttt{GICP}}(\mathbf{T}; \mathcal{R}|\mathcal{X}_t, \mathcal{X}_s, \mathcal{I}) &\triangleq \sum_k^n (||\mathbf{x}_k^t - \mathbf{T} \cdot \mathbf{x}_k^s||_{\mathbf{C}_k}^2) \\ &\propto -\log(p(\mathcal{R}|\mathcal{X}_t, \mathcal{X}_s, \mathcal{I}; \mathbf{T})) \end{aligned}$$

Generalized ICP

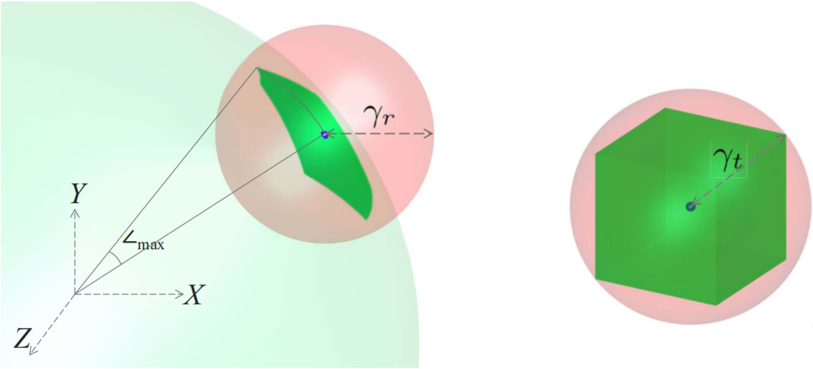
$$-\log(p(\mathbf{r}_k|\mathbf{x}_k^t, \mathbf{x}_k^s, i_k; \mathbf{T})) = ||\mathbf{x}_k^t - \mathbf{T}\cdot\mathbf{x}_k^s||_{\Sigma_k^t + \mathbf{R}\Sigma_k^s\mathbf{R}^\top} + \text{const.}$$

$$\begin{aligned} f_{\texttt{GICP}}(\mathbf{T}; \mathcal{R}|\mathcal{X}_t, \mathcal{X}_s, \mathcal{I}) \;&\triangleq\; \sum_k^n (||\,\mathbf{x}_k^t - \;\mathbf{T}\cdot\mathbf{x}_k^s||^2_{\mathbf{C}_k}) \\ &\propto\; -\log(p(\mathcal{R}|\mathcal{X}_t, \mathcal{X}_s, \mathcal{I}; \mathbf{T})) \end{aligned}$$

$$\underset{\mathbf{T}\in\mathrm{SE}(3)}{\arg\min}\, f_{\texttt{GICP}}(\mathcal{R}|\mathcal{X}_t, \mathcal{X}_s, \mathcal{I})$$

But what if we don't have a good initial guess?

Globally Optimal ICP, Yang et al.



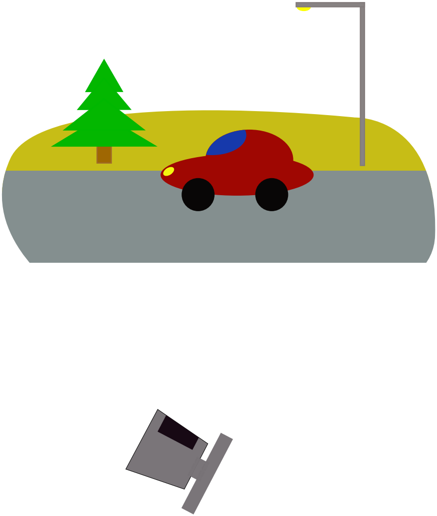
(a) Rotation uncertainty radius (b) Translation uncertainty radius

Correlative Scan Matching, Ed Olson

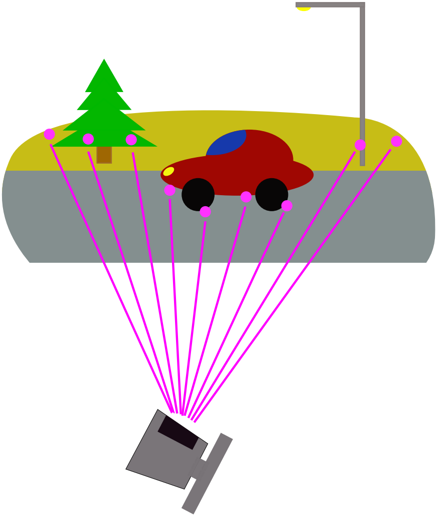
Robust LIDAR Localization using Multiresolution Gaussian Mixture Maps for Autonomous Driving, Wolcott et al.

But why am I telling you about all this?

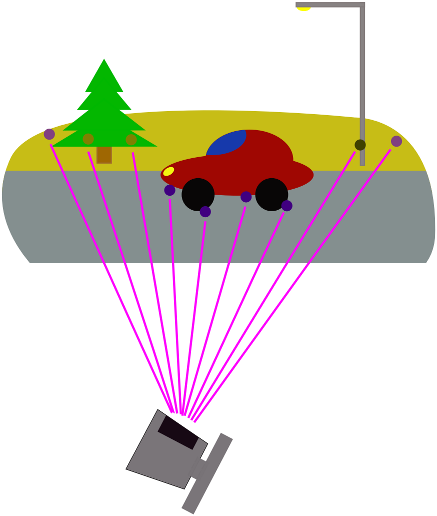
Semantics and Registration



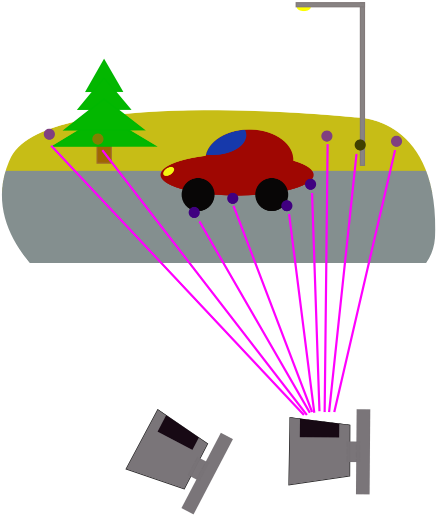
Semantics and Registration



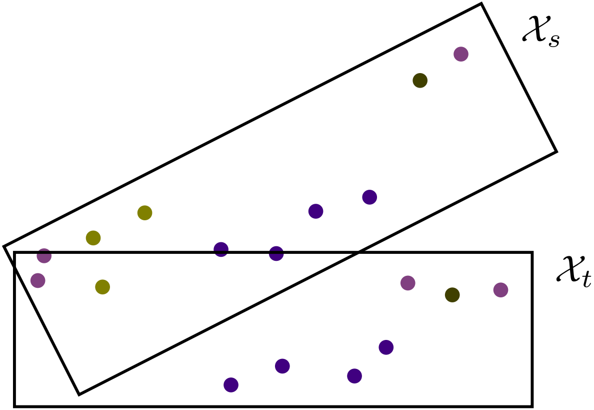
Semantics and Registration



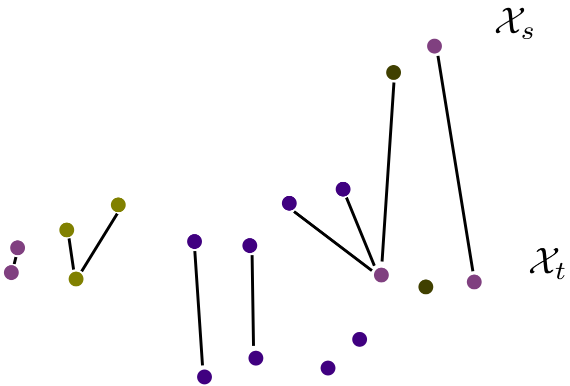
Semantics and Registration



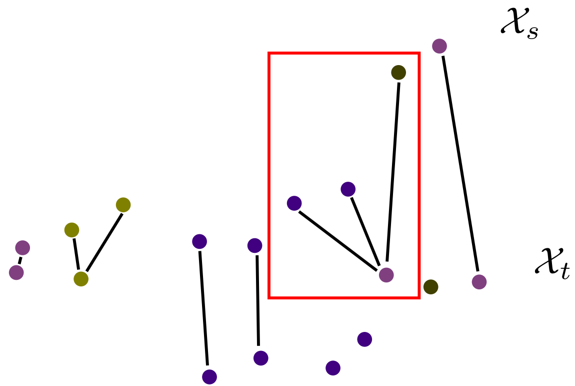
Semantics and Registration



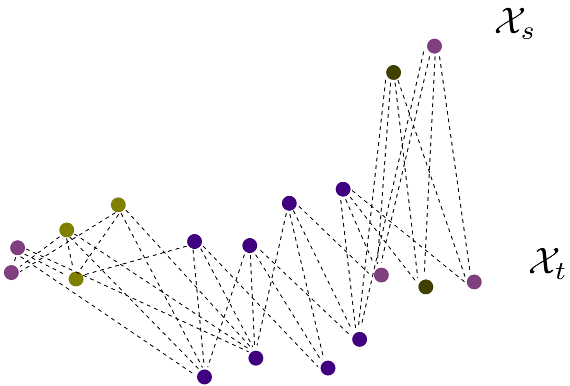
Semantics and Registration



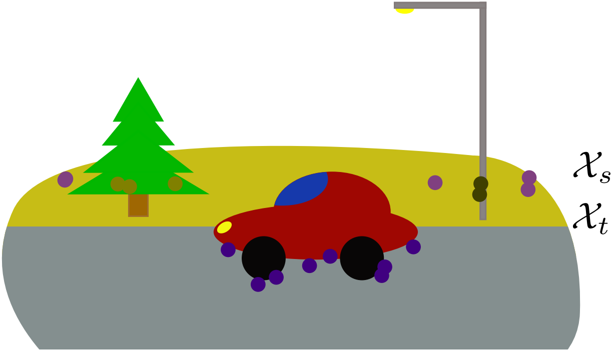
Semantics and Registration



Semantics and Registration

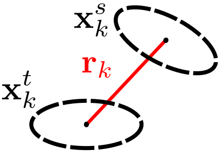


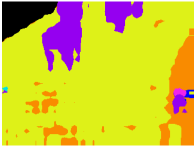
Semantics and Registration



Adding Semantics to Registration

$$p(\mathcal{R},\mathcal{S},\mathcal{I}|\mathcal{X}_t,\mathcal{X}_s) \propto \underbrace{p(\mathcal{R}|\mathcal{I},\mathcal{X}_t,\mathcal{X}_s)}_{\text{residual}} \underbrace{p(\mathcal{S}|\mathcal{I},\mathcal{X}_t)}_{\text{target semantic}} \underbrace{p(\mathcal{S}|\mathcal{I},\mathcal{X}_s)}_{\text{source semantic}} \underbrace{p(\mathcal{I}|\mathcal{X}_t,\mathcal{X}_s)}_{\text{geometric association}}$$





Associations as Latent Variables

- *Expectation:* We wish to compute the expected value of the log-likelihood function with respect to the probability of the association given the current transformation and point clouds, or the $Q(\cdot,\cdot)$ function.

$$\begin{aligned} Q(\mathbf{T},\mathbf{T}^{\mathrm{old}}) &= \mathbb{E}_{p(\mathcal{I}|\mathcal{R},\mathcal{S},\mathcal{X}_t,\mathcal{X}_s;\mathbf{T}^{\mathrm{old}})}[\log p(\mathcal{R},\mathcal{S},\mathcal{I}|\mathcal{X}_t,\mathcal{X}_s;\mathbf{T})] \\ &= \sum_{\mathcal{I}\in\mathbb{I}} p(\mathcal{I}|\mathcal{R},\mathcal{S},\mathcal{X}_t,\mathcal{X}_s;\mathbf{T}^{\mathrm{old}}) \log p(\mathcal{R}|\mathcal{I},\mathcal{X}_t,\mathcal{X}_s;\mathbf{T}) + \text{const.} \end{aligned}$$

- *Maximization:* We wish to maximize $Q(\cdot,\cdot)$ over the transformation variable $\mathbf{T}$

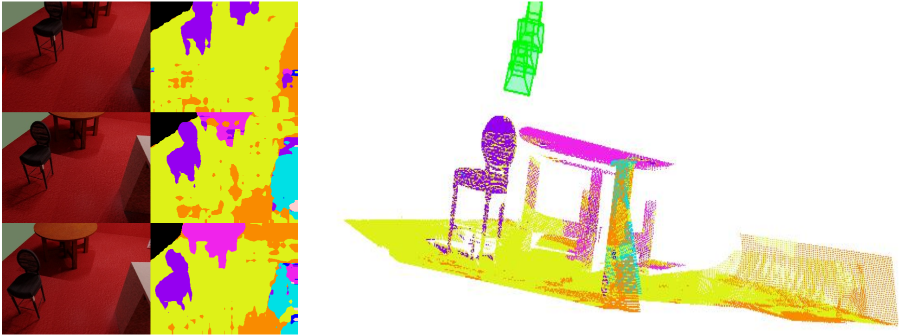
$$\mathbf{T}^* = \arg\max_{\mathbf{T}\in\mathrm{SE}(3)} Q(\mathbf{T},\mathbf{T}^{\mathrm{old}})$$

Semantic ICP

$$w_k \triangleq \sum_{s_k \in \mathcal{C}} p(\mathbf{r}_k | \mathbf{x}_k^t, \mathbf{x}_k^s, i_k; \mathbf{T}^{\text{old}}) p(s_k | \mathcal{X}_t, i_k) p(s_k | \mathcal{X}_s, i_k) p(i_k | \mathbf{x}_k^t, \mathbf{x}_k^s)$$

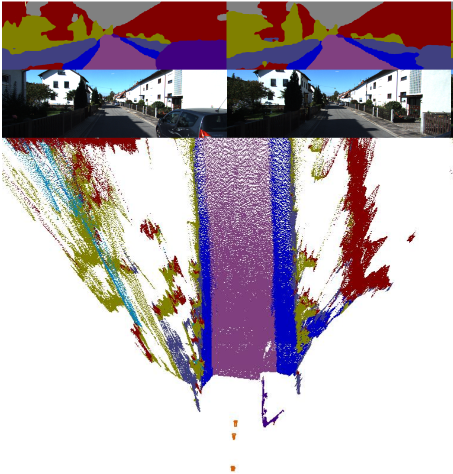
$$\begin{aligned} \mathbf{T}^* &= \underset{\mathbf{T} \in \mathrm{SE}(3)}{\arg\min}\, f_{\texttt{SICP}}(\mathbf{T}, \mathbf{w}; \mathcal{R} | \mathcal{X}_t, \mathcal{X}_s, \mathcal{I}) \\ &= \underset{\mathbf{T} \in \mathrm{SE}(3)}{\arg\min} \sum_{k=1}^{n \times N} \rho_\alpha (w_k || \mathbf{x}_k^t - \mathbf{T}(\mathbf{x}_k^s) ||_{\mathbf{C}_k}^2) \end{aligned}$$

SceneNet RGB-D Results

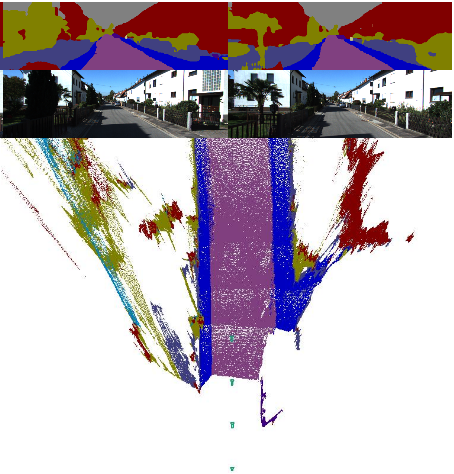


Bed Books Ceiling Floor Furniture Object
Paint Sofa Table TV Wall Window

KITTI Odometry Results



a) GICP



b) Semantic ICP

More resources

Code from this lecture

https://github.com/saparkison/point_cloud_algorithms

Other software libraries

PCL

Open3D

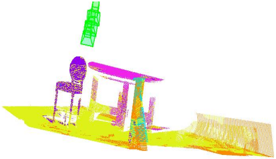
More resources

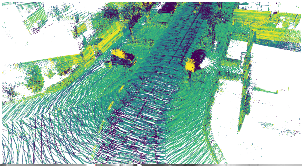
Cyrill Stachniss' Point Cloud Registration Lectures (3 Parts)
Includes the proof of the optimality of the ICP solution.

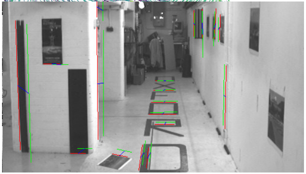
Luca Carlone's RANSAC Lecture.

Thank you!

1. Sensors
2. Point Cloud Algorithms
3. Current Research







Optimization over Matrix Manifolds, Absil et al. 2009

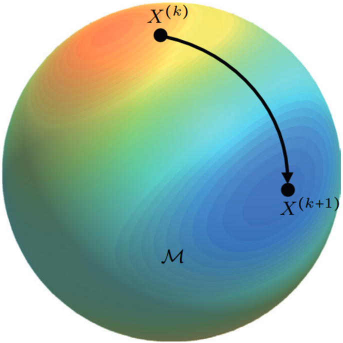


Figure Credit *Michael Bronstein*

Optimization over Matrix Manifolds, Absil et al. 2009

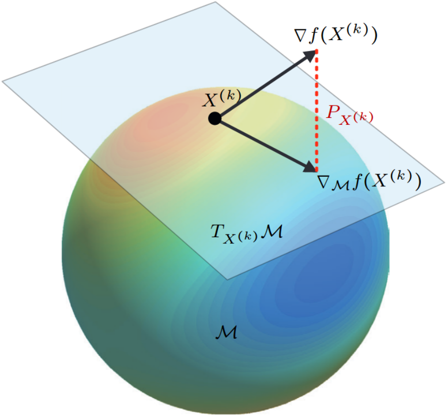


Figure Credit *Michael Bronstein*

Optimization over Matrix Manifolds, Absil et al. 2009

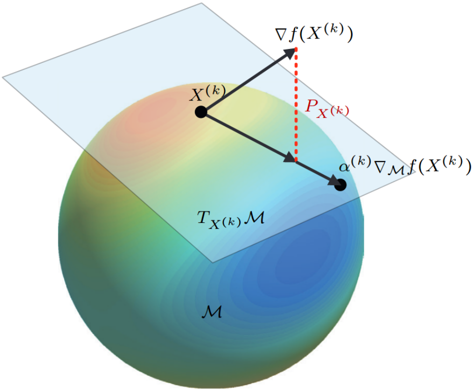


Figure Credit *Michael Bronstein*

Optimization over Matrix Manifolds, Absil et al. 2009

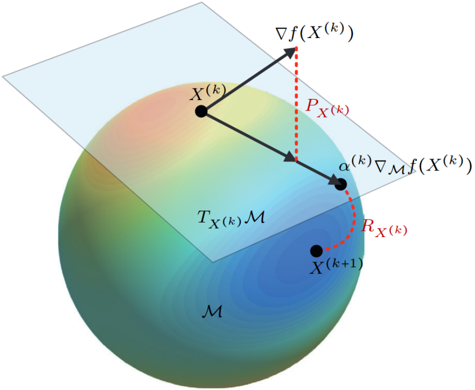
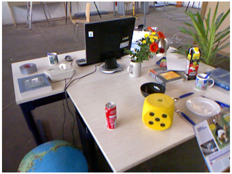
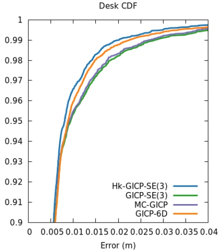
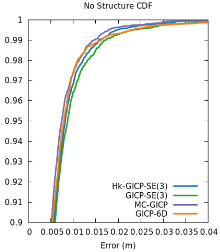


Figure Credit *Michael Bronstein*

TUM RGBD SLAM Results







TUM RGBD SLAM Results

		fr1/desk	fr2/desk	fr3/far	fr3/near
$\mathcal{H}_k$-GICP-$\mathrm{SE}(3)$	mean t (m)	**0.00674**	**0.00137**	0.00391	0.00328
	median t (m)	**0.00517**	0.00075	0.00330	0.00263
	mean r $(^\circ)$	**0.458**	**0.125**	0.210	0.231
	median r$(^\circ)$	0.360	0.068	0.181	0.197
GICP-$\mathrm{SE}(3)$	mean t (m)	0.00775	0.00186	0.00390	0.00344
	median t (m)	0.00529	0.00075	0.00329	0.00265
	mean r $(^\circ)$	0.519	0.144	0.211	0.237
	median r$(^\circ)$	0.372	0.068	0.181	0.199
MC-GICP	mean t (m)	0.00747	0.00154	**0.00371**	**0.00318**
	median t (m)	0.00518	0.00075	**0.00324**	0.00263
	mean r $(^\circ)$	0.492	0.131	**0.202**	**0.229**
	median r$(^\circ)$	0.350	**0.067**	**0.177**	**0.195**
GICP 6D	mean t (m)	0.00677	0.00164	0.00390	0.00325
	median t (m)	0.00521	0.00075	0.00321	0.00263
	mean r $(^\circ)$	0.461	0.134	0.217	0.235
	median r$(^\circ)$	0.360	0.068	0.182	0.199
NDT	mean t (m)	0.02005	0.01074	0.01666	0.01665
	median t (m)	0.005546	0.000755	0.00345	0.002825
	mean r $(^\circ)$	0.893	0.252	0.318	0.590
	median r$(^\circ)$	0.405	0.071	0.217	0.229

Results of evaluation of $\mathcal{H}_k$-GICP-$\mathrm{SE}(3)$ on the TUM RGB-D SLAM dataset in per frame translation error in meters and rotation error in degrees.

MIP Formulaion Handling Outliers

$$\sum_{j=0}^{M} s_{jn} \cancel{=} \geq M - 1$$

$$\sum_{j=0}^{N} s_{mj} \cancel{=} \geq N - 1$$

MIP Formulation Field of View Constraint

$$\frac{u_{\max} - c_u}{f_u}\left(R_3 \cdot p + t_{\max}\right) \geq \left(R_1 \cdot p + t_{\max}\right)$$

$$\frac{v_{\max} - c_v}{f_v}\left(R_3 \cdot p + t_{\max}\right) \geq \left(R_2 \cdot p + t_{\max}\right)$$